THE RHEOLOGY OF THE BLOOD. III*

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Colloidal versus True Solutions

Rheology has not played any very important rôle as yet in either physiology or medicine. This is not because the circulation is unimportant or thoroughly understood. It is rather because of the failure of the simple law of Poiseuille to represent the complex conditions of flow in the blood and other colloidal solutions (1). The invention of the terms "structure viscosity" (2) and "deformation elasticity" (3) seemed to attribute the difficulty to the colloidal character of the solutions requiring a knowledge of the laws of flow of soft solids or plastic bodies. But even a more fundamental mistake has been made in assuming that viscosities are additive in mixtures. In a mixture of simple liquids, the fluidity may not be that given by the additive law in any given case because there are several factors, such as solvation or dissociation which can interfere, but it still remains valid (4) that the fluidities are additive in homogeneous liquid mixtures. In this and succeeding papers we shall, by the use of the literature and our own experiments, attempt to show in a manner not attempted heretofore how the rheological properties of the blood can be usefully correlated. We shall at first regard the blood as a viscous liquid because that is simpler, it is not grossly incorrect, and it enables us to make the maximum use of the earlier work.

In homogeneous mixtures of inert liquids, we assume the fluidities to be additive, and not viscosities, thus

$$\Phi = a\Phi_1 + b\Phi_2$$

or

$$= \Phi_1 + b(\Phi_2 - \Phi_1) \tag{1}$$

where a and b are the volume fractions of the components A and B, whose respective fluidities are Φ_1 and Φ_2. For colloidal solutions, it was found first for suspensions (5) and then for sols that the fluidity (6) is linear still but approaches a value of b which is no longer unity, but b', so that

$$\Phi = (1 - b/b')\Phi_1 \tag{2}$$

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* Bingham, E. C., and Roepke, R. R., The rheology of the blood, unpublished paper presented before the Society of Rheology. The effect of fibrinogen on the fluidity of blood plasma, *J. Am. Chem. Soc.*, 1943, **64**, 1204.

This equation was first applied to clay, graphite, sulfur sol, silver sol, gelatin, and sodium palmitate, etc. Treffers (7) has extended the principle of additive fluidities to various protein solutions. He concluded that "in addition to its advantages of simplicity and ease of application, it fits available data quite well .... The fluidity is linear with the protein concentrations over a wide range of proteins and concentrations."

Absolute versus Relative Fluidities

In giving data as absolute fluidities in rhes, an explanation is required for the departure from the usual custom of using *relative* values only. (1) The use of relative fluidity, defined as the ratio of the fluidity of blood to the fluidity of water at 20°, would not give a quantity which can be operated upon mathematically. (2) W. R. Hess (8) who was largely responsible for the adoption of the use of "relative viscosity" did not use water at a fixed temperature as standard, but rather at *the temperature of measurement.* This adds the disadvantage that the standard is not a fixed standard and to a degree the standard is uncertain. The example of Hess has been generally followed. (3) Hess (8) claimed that the relative fluidity as defined by him is independent of the temperature. The result has been that the word "relative" is often omitted from the data and the temperature of measurement is not mentioned. The claim of Hess may be true, or nearly true, but the use of relative fluidities would tend to conceal its significance, for two different liquids cannot be assumed *a priori* to have the same temperature coefficient of fluidity. Thus for example, water and mercury have the same fluidity of 60 rhes at 2.1° C. but at 20°C. the fluidity of mercury is 64.3 while the fluidity of water is 99.5 rhes, so that the relative fluidity at 20° is no longer unity but 0.646. The answer to the question as to why the Hess assumption could be made for blood has never been given, but of that later (p. 84). (4) Actually Hess recommended the measurement of the viscosity of the blood at room temperature as a matter of convenience, but the physiologist is primarily interested in the blood at its "working temperature." For example, Burton-Opitz (9) recognized that the temperature of the living blood is the temperature at which to compare different animals and then absolute fluidities become significant, the frog with the low working temperature having approximately the same fluidity of blood as the mammal at 37°. (5) But this virtual disregard of the temperature in recording only relative fluidities would virtually assume as a fact that different samples of blood are alike, so that cooling one blood down to room temperature might actually have an effect somewhat different from that on the ordinary or normal blood. It is not necessary to be specific as to whether the result is a coagulation, rouleaux formation, or crystallization. It seems an unnecessary complication to be avoided if possible. (6) Finally, in tables

of constants, it is confusing to have data based on several standards, particularly since we do not have even one standard of unquestioned accuracy, water at 20°C. $\Phi = 99.5$ rhes.

Nevertheless, many data in the literature have been converted to fluidities and compared. We will study first the data of Nägeli (10) on the viscosity of human blood serum. The data as taken from Bircher (11*a*) are given in Table I, but the obviously incorrect value of 1.57 has been changed to 1.51 as a probable typographical error. Since the measurements were made at about 20°, the fluidity has been calculated for that temperature (column 3); but we have also calculated the fluidity of the blood serum at 37° (column 4),

TABLE I

The Relation of the Fluidity of Human Blood Serum to the Protein Content

After Nägeli (10). $\Phi_{20°} = 89.8 - 3.99\ b$; $\Phi_{37°} = 129.6 - 5.72\ b$

Protein	Relative viscosity	Fluidity at 20°	Fluidity at 37°	Fluidity at 20° equation (2)	Fluidity at 37° equation (2)
per cent		*rhes*	*rhes*		
5.0	1.43	69.6	100.7	69.8	101.0
5.5	1.46	68.2	98.6	67.8	98.2
6.0	1.51	65.9	95.4	65.8	95.3
6.5	1.56	63.8	92.3	63.8	92.4
7.0	1.61	61.8	89.4	61.9	89.6
7.5	1.67	59.6	86.2	59.9	86.7
8.0	1.72	57.8	83.7	57.9	83.8
8.5	1.78	55.9	80.9	55.9	81.0
9.0	1.84	54.1	78.3	53.9	78.1
9.5	1.90	52.4	75.8	51.9	75.3

on the assumption of the relative viscosity (or fluidity) being independent of the temperature. Applying equation (2) to these data, the constants b' and Φ_1 have been obtained and the fluidities given in the last two columns (5 and 6) of the table computed. They agree well with the "observed" values of columns 3 and 4 respectively the average deviation being nearly 0.3 per cent for each case. The fluidity curves are therefore linear. At 20° the value of b where $\Phi = 0$ is 0.225 and at 37° it is 0.226, which may be considered identity, but the fluidity of the medium at zero concentration of protein turns out to be $\Phi_1 = 89.8$ rhes at 20° and 129.6 rhes at 37°C. The non-protein medium has a fluidity considerably lower than water, 99.5 at 20° and 144 at 37° the difference amounting to nearly 10 per cent.

For a second study, it is interesting to use data measured by Hess (13) in 1906 obtained by measuring the fluidities of mixtures of human blood serum with different amounts of physiological salt solution. He obtained a strongly hyperbolic curve which was confirmed with the serum of other men and ani-

mals. He was apparently puzzled, because others working under his direction reverted to the same problem (Blunschy (14) and Bircher (11*b*)) and they discussed the fact that the curve is non-linear. It did not occur to them to use fluidities which are linear, as shown in Table II. The experiments of Bircher (11) with sheep blood serum are more precise and numerous, hence

TABLE II

The Fluidity of Human Blood-Serum and Physiological Salt Solution

After Hess (13). $\Phi_{37°} = 115.8 - 0.936\ b$

Volume	Relative viscosity	Fluidity at 37°	Fluidity at 37°, equation (2)
per cent		*rhes*	
20	1.4	102.8	97.1
40	1.8	80.0	78.4
60	2.75	52.4	59.7
80	3.8	37.9	41.0
100	5.7	25.3	22.2

TABLE III

The Relation of Fluidity to the Protein Content of Sheep Blood Serum

After Bircher (11)

Volume of serum	Protein	Bircher relative viscosity corrected to 15°	Fluidity at 20° observed	Fluidity at 20° calculated
	per cent		*rhes*	*rhes*
10	0.7	1.05	95.2	94.3
20	1.4	1.10	90.1	90.4
30	2.1	1.16	86.2	86.5
40	2.8	1.22	82.6	82.5
50	3.5	1.28	78.4	78.6
55	2.85	1.32	76.3	76.7
60	4.2	1.35	74.1	74.7
65	4.55	1.405	72.5	72.8
70	4.9	1.435	70.9	70.8
80	5.6	1.53	66.7	66.9
90	6.3	1.63	63.3	63.0
100	7.0	1.74	59.5	59.0

they have been employed in Table III. Bircher used the serum of several different animals which he mixed with 0.95 per cent saline solution. The fluidities have been computed from his data for 20° and the formula fitted to the data is $\Phi = 98.2 - 5.60\ b$, where b is the fraction of protein present. With $\Phi_1 = 98.2$, which is the value expected for physiological salt solution, the concentration required for zero fluidity; *i.e.*, $b' = 0.176$. The percentage deviation given by the formula is 0.4 per cent.

TABLE IV

The Fluidity of Mixtures of Human Blood Plasma with Sediments in Plasma Containing 6,616,000 Erythrocytes per mm³.

After Blunschy (14). $\Phi = 63 - 52.3\ b$

Volume	Relative viscosity 19°	Fluidity at 37° observed	Fluidity at 37° calculated
per cent		*rhes*	*rhes*
0	2.27	63.5	63.0
25	2.87	50.1	50.2
50	3.92	36.7	37.2
75	6.25	23.0	24.3
100	11.4	12.6	11.4

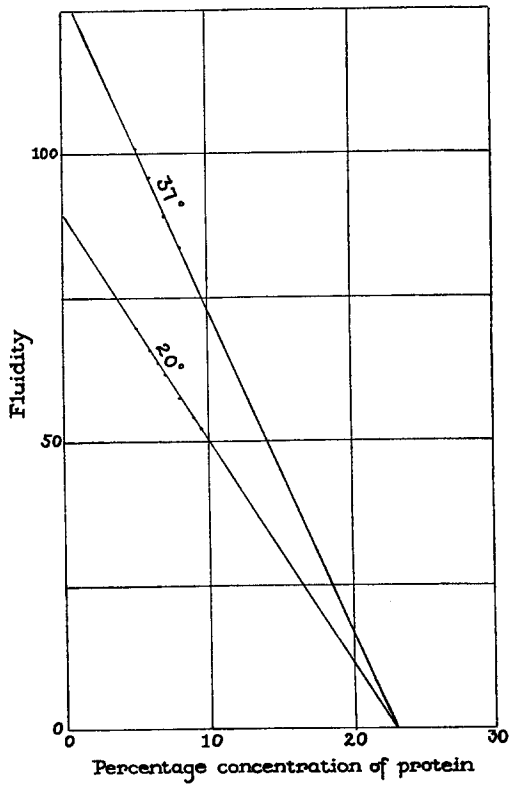

FIG. 1

Having shown that equation (2) applies to many types of colloidal solutions including proteins and even blood serum, it is desirable to include data (Table IV) from Blunschy, page 10, on suspensions of human red blood corpuscles in plasma. The plasma was obtained by centrifuging and the mixtures made

by the addition of the plasma to a sediment containing 6,616,000 corpuscles per mm.[3] The measurement was made at 19° and the blood was venous. The fluidity of the plasma at 37° is $\Phi_1 = 63.0$ and the concentration of zero fluidity is $b' = 1.02$, which signifies that the sediment contained enough plasma so that it behaved as a viscous liquid; nevertheless the sediment gives a fluidity which is 12.6, which is much too far away from the calculated value of 11.4 rhes. The average percentage deviation is only 0.7. In passing it is worth noting that this blood was taken from a patient suffering from croupous pneumonia and may therefore have had a considerable tendency to coagulate, with a high sedimentation rate. These results which seemed inexplicable

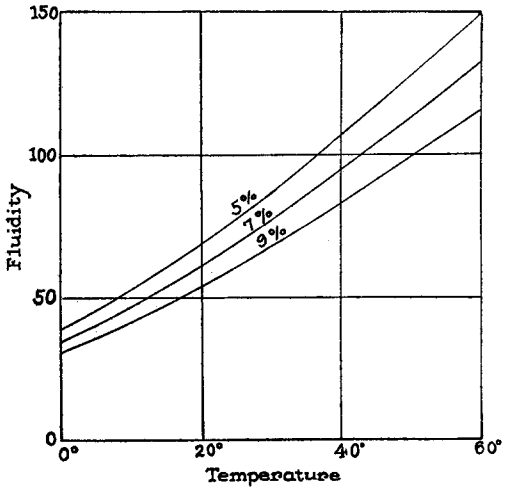

FIG. 2

to Blunschy, seem normal enough now. But if a sediment with much less plasma were used, we may be sure that problems of plastic flow would enter.

The Effect of Temperature on the Fluidity of Protein Solutions

We now revert to the consideration of the rule of Hess that the relative fluidity (or viscosity) of blood is independent of the temperature, at least within certain specified limits but they have often been disregarded. The fluidity of human blood serum as given by Nägeli for various concentrations of protein in Table I has been plotted in Figs. 1 and 2, the fluidity *versus* concentration and temperature respectively. The linear character of the fluidity concentration curve is a demonstration of the applicability of the rule of Hess to human blood serum: it is also in accordance with the law of fluidity of colloidal suspensoids, equation (2), and we get

$$\frac{\Phi_{t_1} c_0}{\Phi_{t_2} c_0} = \frac{\Phi_{t_1} c_1}{\Phi_{t_2} c_1} \tag{3}$$

where $\Phi_{t_1 c_0}$ represents the fluidity at temperature t_1 at zero concentration, etc., and according to the rule of Hess

$$\frac{d\Phi}{db} = -[K]_t = \text{concentration coefficient of fluidity,}$$

hence on integration,

$$\Phi = C - [K]_t b \tag{2a}$$

where $C = [\Phi]_{c=0}$, *i.e.*, the fluidity of the medium when the concentration is zero. Also $[K]_t = \Phi_1/b'$.
From equation (3) it follows that

$$\frac{\Phi_{t_1 c_1}}{\Phi_{t_1 c_0}} = \frac{\Phi_{t_2 c_1}}{\Phi_{t_2 c_0}} = [K]_b$$

then

$$\frac{d\Phi}{dt} = [K]_b = \text{temperature coefficient of fluidity}$$

and

$$\Phi = [K]_b t + D \tag{4}$$

where D is the fluidity at temperature zero, $[\Phi]_{t=0}$ and $[K]_b = -[\Phi]_{t=0}/t'$ As the temperature is lowered, t' is the temperature at which the fluidity becomes zero.

One cannot assume that the above relation will apply exactly in a given case. The law is valid for the ideal case, and therefore it is the deviations from the law that engage the greatest interest. The law itself merely states that the fluidity of a suspension depends solely upon the fluidity of the volume of medium which is present. This suggests at once that if some of the medium is immobilized by being absorbed into porous particles or if the particles are partially dissolved in the medium or if aggregates are broken down on shearing, the application of the law will show deviations. This matter is of such importance that a case in point will now be considered, using the abundant data of Jacques Loeb (15) on isoelectric gelatin up to 4 per cent gelatin[1] at temperatures from 25 to 60°.

We have converted the relative viscosities to rhes given in Table V and we have plotted the data in Fig. 3. Loeb noted that "at 25° the agreement is satisfactory only at the lowest concentrations.... The gelatin solidifies so rapidly that viscosity measurements were no longer possible for a concentration of 3.5 per cent." The curves are all linear and all except the data for 25° converge at −74 rhes, and in view of Loeb's remarks we reserve any

[1] Loeb, page 204, gives the data under the wrong heading "log η/η_0" instead of "η/η_0" in the original paper, *J. Gen. Physiol.*, 1919, **1**, 483.

TABLE V

The Fluidities of Solutions of Isoelectric Gelatin in Water at Various Concentrations and Temperatures

After J. Loeb and calculated by the formula: $\Phi = (1 - 0.0659\, b)\,(\Phi_1 + 73.9) - 73.9$

Concentration	$\Phi_{25°}$ Observed	$\Phi_{25°}$ Calculated	$\Phi_{35°}$ Observed	$\Phi_{35°}$ Calculated	$\Phi_{45°}$ Observed	$\Phi_{45°}$ Calculated	$\Phi_{60°}$ Observed	$\Phi_{60°}$ Calculated
vol. per cent								
0	(111.9)	110.3	(138.4)	135.1	(167.0)	163.1	(213.3)	212.1
0.25	107.9	107.3	134.8	131.8	162.0	159.3	208.4	207.6
0.50	103.7	104.4	129.6	128.5	156.3	155.6	203.1	203.0
1.0	95.8	98.1	120.6	121.3	147.1	147.5	195.2	193.3
1.5	87.7	92.7	111.9	115.1	137.6	140.4	183.0	184.8
2.0	81.7	86.8	106.0	108.5	130.5	132.9	172.7	175.7
2.5	76.2	81.0	100.2	101.8	123.6	125.4	164.7	166.6
3.0	66.1	75.1	93.3	95.2	115.9	117.8	162.9	157.4
3.5	—		89.2	88.5	111.0	110.3	149.0	148.4
4.0	—		86.3	81.8	106.6	102.7	140.2	139.2
Average deviation, *per cent*.....			2.2		1.5		0.9	

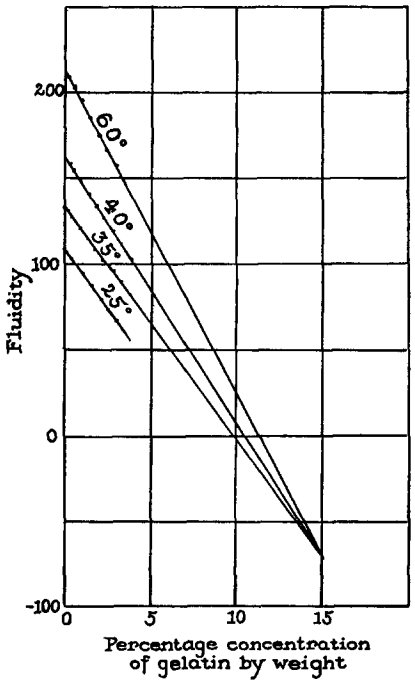

FIG. 3

judgment in regard to that exception. The fact that the curves differ from all suspensions heretofore studied in not converging at zero fluidity signifies that the temperature of zero fluidity falls as the temperature of the gelatin solutions studied is lowered, and it can be explained, since gelatin is a lyophilic colloid and known to form hydrates, by the gelatin absorbing an increasing amount of water as the temperature is lowered. This necessitates adding a parameter to equation (2), which becomes

$$\frac{\Phi - c}{\Phi_1 - c} = 1 - 0.0659b \qquad (2b)$$

where $c = -73.9$. The fluidities calculated by this for the various temperatures and concentrations are all given in the table. The average deviation (1.5 per cent) between the observed and calculated fluidities increases some-

TABLE VI

Constants for 35°, 45°, and 60°, in Equation (2c) for Isoelectric Gelatin

Temperature	Constants			
	α observed	α calculated	β observed	β calculated
°C.				
35	135.1	138.4	13.35	10.12
45	163.1	167.0	15.21	10.73
60	212.1	213.3	18.06	11.74

what as the temperature is lowered. In obtaining the equation (2*b*) we first obtained the best linear equation of the form of equation (2)

$$\Phi = \alpha - \beta b \qquad (2c)$$

These constants are given in Table VI together with the fluidity of water at the same temperatures. The lowest volume percentage of gelatin is 0.25 and it gives no indication of lack of linearity, yet this may be best explained by a portion of the gelatin dissolving in the water to form a true solution, thereby lowering the fluidity.

As the value of β in equation (2*c*) or Φ_1/b' in equation (2) is a function of the temperature, it seems highly desirable for our further study in human physiology to have a standard temperature for the comparison of all proteins, 37° being very convenient as well as logical. In the case of gelatin $\Phi - c$ is used in place of Φ in equation (2*b*) and thus a value for β agreeing closely for the three temperatures of 35°, 45°, and 60° is obtained of 13.72. This is then a fairly definite and reproducible rheological constant for proteins and perhaps other types of colloids. Since the rheological properties of colloids vary

within wide limits and are very important as well as characteristic, such a constant may be of value.

Before leaving the consideration of gelatin, it is noted that the concentration of zero fluidity rises with the temperature and therefore at some concentration may reach unity; equation (2) then becomes

$$\Phi = (1 - b)\Phi_1$$

and since $1 - b = a$
therefore

$$\Phi = a\Phi_1 \qquad (2d)$$

This critical equation is so called because it is the transition between true and colloidal solutions, between equations (1) and (2). It is the analog of the law

TABLE VII

The Fluidity-Temperature Coefficient of Whole Blood

After Kagan (12)

η relative 37° observed	η relative 17° observed	Φ 37° observed	Φ 17° observed	Φ 17° calculated
3.8	4.6	37.9	31.3	31.4
4.0	4.85	36.0	29.7	29.8
3.08	3.7	46.8	38.9	38.8
4.25	5.1	33.9	28.2	28.1

of Raoult. True solutions are viscous whereas at shearing stresses below the yield stress, colloidal solutions are plastic.

The Fluidity-Temperature Coefficient

Kagan (12) made measurements of the relative viscosity of whole blood at both 17° and 37° from four individuals with the special purpose of checking the statement of Hess that this lowering of the temperature of the blood caused a 16 per cent increase in the relative viscosity or 0.8 per cent per degree. We give the scanty data in Table VII for what they are worth realizing that measurements at three or more temperatures are necessary if we are to learn how the fluidity varies with the temperature. The observed relative viscosities are given in the first two columns and the computed fluidities in rhes in the third and fourth columns, and the calculated fluidities at 17° in the fifth column, using the following formula:

$$\Phi = (1 - \gamma\Delta t)\Phi_{17} \qquad (2e)$$

which is a form of equation (2), where γ is the fluidity-temperature coefficient per degree from 37°, the temperature difference being $\Delta t = 20°$. From the

observed fluidities, the average value of γ is found to be 0.86 instead of simply 0.8 and the agreement between the observed values and those calculated by the formula are all that can be expected from the data.

In an ideal suspensoid in water, the fluidity-temperature coefficient should be a fraction of that of the medium itself, which can be best obtained by differentiation of the fluidity-temperature equation. But we may say that calculated in the same way over the same temperature interval, water gives a value for the fluidity-temperature coefficient which is 1.79 which is double the value accepted for blood. The obvious explanation, that the medium in the case of blood is not pure water, is hardly satisfying. The question needs looking into. Fig. 2 proves that the fluidity-temperature curves are not linear.

The Fluidity of a Mixture of Proteins

If the fluidity of a mixture of protein solutions is to be calculated from the concentration coefficients of the individual proteins, more information is required than may be available at present. Many of the data now available are for one temperature only, so that the estimation of the hydration is not practicable. Fortunately, the blood proteins are apparently but little hydrated. For example, Kunitz, Anson, and Northrop (16) report that the hydration of gelatin is forty-five times greater than that of hemoglobin. We can, however, calculate the concentration coefficient for the temperature of the blood $\beta_{37°}$ on the assumption that equation (2) applies, which has been done for over twenty proteins (Table VIII). There are two difficulties further, the one that concentrations are often based on weight instead of volume; another embarrassment is due to the possibility that the solvent medium is not pure water. This becomes a certainty in the cases of globulin and fibrinogen, since they dissolve in salt solutions only, but a physiological salt solution has a fluidity of only about 1 rhe below that of water. And it goes without saying that the pH of the medium needs careful consideration. Table VI shows that the average deviation between the values of α and Φ_1 is only 1 per cent. Dissolved substances would nearly all decrease the fluidity but the lowering exceeds the raising of the fluidity by only 0.1 per cent which is indeed negligible. The values of $\beta_{37°}$ are arranged in decreasing order.

If in the blood the various salts and proteins are present in concentrations, b_1, b_2, b_3, etc. with values of the concentration-coefficients of β_1, β_2, β_3, etc., then on the hypothesis that fluidities are additive

$$\Phi = \Phi_1 - \beta_1 b_1 - \beta_2 b_2 - \beta_3 b_3 - \cdots \tag{5}$$

or the lowering of the fluidity of the blood at 37° $\Phi - \Phi_1 = \Delta\Phi$
and

$$\Delta\Phi = \beta_1 b_1 + \beta_2 b_2 + \beta_3 b_3 + \cdots \tag{6}$$

where the first term on the right side represents the total effect of the salts 1.3 rhes, the second term that of the albumin of perhaps 24.4 rhes, the third that of globulin, the two together amounting to perhaps 56.2 rhes and the fourth that of fibrinogen, 16.2 rhes. The average values for the proteins in human blood given by Lewinski (17) are 4.01 serum albumin, 2.83 serum globulin,

TABLE VIII

The Fluidity-Concentration Coefficients ($\beta_{37}°$) for Various Protein Solutions

Protein	α	Φ	β x 100	$\beta_{37°}$ x 100
Globulin (20)	115	114	2237	2801
Globulin (beef) (20)	117	114	785	965
Fibrinogen (dog)*	112	112	26.4	33.9
Globulin (horse) (21) III	114	112	23.8	29.8
Globulin (horse) (21) II	115	112	16.6	20.8
Globulin (horse) (21) I	113	112	11.6	10.43
Globulin (horse) (21) sample 2	109	112	10.4	13.77
Gelatin, isoelectric (14)	135	138	13.4	13.72
Gliadin (22)	99	99.5	9.93	13.02
Pseudoglobulin (21)	112	112	8.99	11.60
Octopus hemocyanin (22)	97.6	99.5	5.39	7.95
Trypsin (16)	65.7	65.8	3.30	7.23
Albumin (horse) (21)	111.7	111.9	5.13	6.61
Albumin, sample 2	110.4	111.9	4.80	6.26
Amandin (22)	98.5	99.5	4.38	6.40
Serum albumin (22)	98.7	99.5	3.94	5.76
Serum albumin (10)	130	144	5.72	5.72
Lactoglobulin (22)	98.5	99.5	3.76	5.50
Ovalbumin (22)	98.9	99.5	3.76	5.49
CO-Albumin (22)	99.3	99.5	3.75	5.43
CO-Albumin (16)	66.1	65.8	2.40	5.24
Egg albumin (14)	87.8	87.6	1.41	2.32
Oxyhemoglobin (24)	112.4	111.9	1.80	2.28

* Bingham, E. C., and Roepke, R. R., The rheology of the blood, unpublished paper presented before the Society of Rheology. The effect of fibrinogen on the fluidity of blood plasma, *J. Am. Chem. Soc.*, 1943, **64**, 1204.

and 0.42 fibrinogen, reckoned as grams per 100 cc. of plasma. From the above $\Delta\Phi$ turns out to be 73.7 rhes and therefore the fluidity of blood plasma would be 144.0 − 73.7 = 70.3 rhes. This accords well with the average value.

We have not yet considered the effect of erythrocytes, leukocytes, and platelets on the fluidity of the blood. That subject will be considered in the next paper. The plasma and serum follow the law of Poiseuille without question so exact constants can be obtained, at least in theory. But with albumin, globulin, and fibrinogen qualified by the source and perhaps other conditions,

the identity of the substances studied here appears not well defined, hence much work is needed in confirming the earlier measurements. To give added point, it is noted that the data on protein solutions of a few observers deviate widely from the linear curve. We have not been able to fit a linear curve to the data by Chick (18) for pseudoglobulin or for egg albumin by Chick and Lubrzynska (19). This seems less important since other data on these same substances by different observers do yield linear fluidity-concentration curves. It is hoped that these irregularities may be explained.

CONCLUSIONS

1. The fluidity-concentration equation (1) for true solutions and the corresponding equation (2) for suspensoids and suspensions merge into each other in equation (2*d*), $\Phi = a\Phi_1$, where Φ_1 is the fluidity of the medium A, of which a is the volume fraction present. Equation (2*d*) defines the critical relation between colloidal and true solutions and is the analog of Raoult's law.

2. The importance of not only using fluidities but also of using absolute fluidities (rhes) and not relative fluidities is emphasized.

3. The rule of Hess, that the fluidity (or viscosity) of blood, serum, or plasma is independent of the temperature was based on observation but without theoretical explanation. The above law of fluidity of suspensoids that the fluidity-concentration curves are linear, coupled with the observation that the concentration at which the fluidity reaches zero is independent of the temperature, makes the rule of Hess follow as a necessity. The rule is true because the fluidity of a suspensoid is due solely to the volume of the medium present and its coefficient of fluidity. Therefore it is possible to predict the fluidity of such a solution at any concentration or temperature, from a knowledge of b' and that of the fluidity of the medium at the desired temperature. These relations follow as a result of the fact that the fluidity-temperature curve of water is *nearly* linear. The rule of Hess is known to be only approximately true and the same may be true of equation (2).

4. If the value of b' is a function of the shearing stress, it is probably because the material is plastic above a certain concentration and below a certain shearing stress—the yield stress. No plasticity has been detected by us in the blood plasma at the temperature and concentration of proteins found in the body. This however does not apply to whole blood.

5. With lyophilic colloids, it is found that the value of b' changes with the temperature of the solution, which is attributed to changes in solvation with the temperature. By the use of one additional parameter, it is nevertheless possible to still predict the fluidity of solutions of isoelectric gelatin from 35 to 60° with an average deviation of less than 2 per cent.

6. The fluidity of the various proteins in the plasma can be calculated from the fluidity-concentration coefficients at 37° which is a convenient standard

temperature. These coefficients can be calculated from existing data provisionally on the reasonable assumption that most of the proteins in blood, albumin, globulins, and fibrinogen are not greatly hydrated. If the fluidities of mixtures of these proteins are additive, *i.e.*, compatible, it becomes feasible to calculate the fluidity of the plasma from the fluidity-concentration coefficients and the fluidity of water, 144.0 at 37°. Whereas the result is reasonable, the constants are only tentative and they are to be used with caution.

SUMMARY

The authors have confirmed the fact that blood serum and plasma behave rheologically like a true viscous liquid. It is true for whole blood only to a first approximation, but with this reservation they have studied the available data and extended the equation of Bingham and Durham to cover protein solutions of various concentrations and at various temperatures as well as mixtures of proteins and corpuscles present in whole blood. If Φ is the fluidity of whole blood, Φ_1 is the fluidity of water and $\Delta\Phi = \Phi - \Phi_1$, then

$$\Delta\Phi = \beta_1 b_1 + \beta_2 b_2 + \beta_3 b_3 + \cdots$$

where β_1, β_2, β_3, etc., are constants for the fluidity lowering of the salts, albumin, globulin, fibrinogen, and the corpuscles, etc., present in the whole blood.

The conclusions from the data referred to are intended to buttress this simple equation (6).

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