INTERPRETATION OF SURFACE ASPECTS OF CELL SECTIONS

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INTRODUCTION

In many types of cells, portions of the cellular surface are specialized. The morphology of such areas often reflects this specialization: the cell membrane may form ciliae, villi, etc.; or cytoplasmic processes such as pseudopods may protrude. It would be of great interest to know the probability for one random thin section of a cell to demonstrate the existence of such a specialized area. More generally, given a population of structures presenting on a portion of their surface a particular character, we would like to define the probability for one random section in such a structure to show this character.

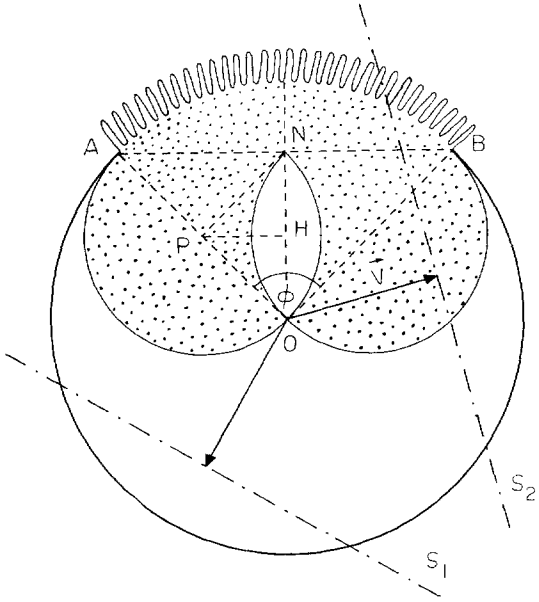

FIGURE 1 Two-dimensional representation of a spherical structure presenting a morphological particularity on a portion of its surface. AB, arc corresponding to the cross-section of the cap of the sphere presenting the characteristic; $OA = OB = R$; ϕ, angle defining the cap; **v**, vector defining the secant planes. S_1 and S_2 are two-dimensional representation of secant planes which intersect (S_2) or do not intersect (S_1) the cap. The dotted surface represents the cross-section of the volume generated by the extremity of the vectors **v** corresponding to the secant planes which reach the cap.

METHOD

We assume that: (*a*) the structure is a sphere of radius R; (*b*) the morphological particularity is located on a cap of the sphere, defined by the solid angle ϕ; (*c*) the probability for one random section through the structure to show the particularity is given by the ratio of the number of the secant planes intersecting the cap to the total number of planes intersecting the sphere.

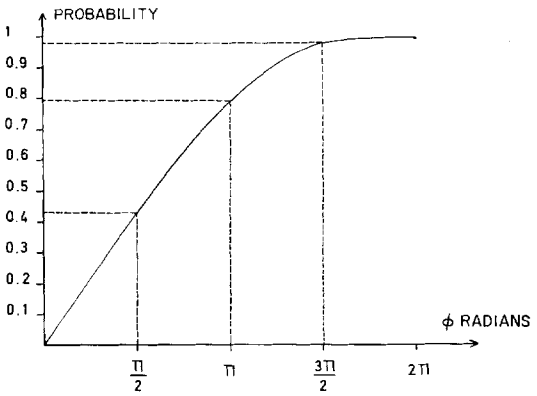

FIGURE 2 Plot giving the probability for one secant plane chosen at random to intersect a cap defined by the solid angle ϕ of a sphere. It gives the probability for one thin section of a cell to show a morphological particularity located on a portion of the cellular surface corresponding to the extent of such a cap. The angles are measured in radians.

To each secant plane of the sphere, we associate the perpendicular vector **v** (the modulus of which is smaller than R) arising from the center O of the sphere. If **v** scans the whole sphere in any direction, the extremities of the vectors corresponding to secant planes which intersect the cap generate a volume V_1. This volume divided by the volume of the sphere

TABLE I

Some Key Values of the Probability for One Secant Plane Chosen at Random to Intersect a Cap Defined by the Solid Angle ϕ of a Sphere

ϕ	5°	10°	20°	40°	60°	90°	120°	180°	270°
Probability	0.03	0.05	0.10	0.20	0.30	0.43	0.56	0.79	0.98

For convenience, ϕ is here measured in degrees.

(generated by the extremities of the vectors corresponding to all the secant planes) gives the probability for the cap to be intersected by a secant plane chosen at random.

Such a system possesses an azimuthal symmetry so that we can represent it in a two-dimensional space by a cross-section. Fig. 1 shows such a section. We compute the fraction of the spherical volume V_1 that the shaded area on the cross-section would occupy if rotated in azimuth through an angle of π.

$$V_1 = \pi R^3(\tfrac{1}{3}[1 - \cos\alpha][2 - \cos\alpha - \cos^2\alpha] + \tfrac{1}{4}\sin\alpha[\pi - \alpha + \sin\alpha\cdot\cos\alpha]),$$

$$\text{Probability} = \frac{V_1}{4/3\cdot\pi R^3} = \frac{1}{2} + \frac{3}{16}(\pi - \alpha)\sin\alpha - \frac{9}{16}\cos\alpha + \frac{1}{16}\cos^3\alpha,$$

where $\alpha = \phi/2$.

RESULTS AND APPLICATION

Fig. 2 shows a plot of probability vs. ϕ. Some key values for probability are given in Table I. This formula can be applied to sections of any subcellar, cellular, or pluricellular spherical structures (e.g. renal glomeruli). We have applied this method to the following example. In electron microscope study of isolated sheep thyroid cells, Nève et al. (1) (later supported by Tixier-Vidal et al. [2]) observed cells widely separated from each other; no brush border was observed on 155 of these cells although it is well known that thyroid cells *in situ* possess such a structure on their apical border. If we assume that a thyroid cell is cubical *in situ* and becomes spherical when isolated (as suggested by Nève et al. [1] and Tixier-Vidal [2]), the ϕ corresponding to one face of the cube transported on the sphere equals $(\pi/2)$ radians (90°). Since the thickness of the sections is small (about 0.05 μ)[1] as compared with the maximum observed diameter of the cells (8μ),[1] we are allowed to apply the formula. So probability = 0.43 (Fig. 2). If the cells exposed to microtome section have no preferential orientation, these authors should have observed approximatively 66 cells (155 × 0.43) showing a brush border. The probability of not observing any cells with a brush border in the population would have been in the range of 10^{-38}. This calculation thus supports the conclusion that these isolated cells had lost their brush border.

From a general point of view, it is worthwhile to note: (*a*) that the same method with only slight modifications in the calculus would allow computing such probabilities in any structure presenting a cylindrical symmetry; (*b*) that, if the frequency of demonstration of a surface character on sections is known (e.g. from micrographs), it is possible to compute the extent of the surface presenting the character.

[1] P. Nève. Personal communication.

SUMMARY

Given a population of isolated structures presenting on a portion of their surface some morphological particularities, a mathematical model is developed which allows computation of the probability for one thin section in such a structure to show this particularity. An example of application of the method is given in the case of isolated thyroid cells.

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