A NEW VISCOMETER.

By P. LECOMTE DU NOÜY.

(*From the Laboratories of The Rockefeller Institute for Medical Research.*)

(Received for publication, December 11, 1922.)

The problem of measurement of viscosity has been dealt with by a number of investigators since the beginning of the 19th century, and the laws governing the flow of fluids in general have been established on a firm mathematical basis. In a recent publication, Bingham[1] reviews the actual state of the question. However, when it comes to the measurement of the viscosity of small quantities of liquids with great accuracy, as great as the control and measurement of temperature will permit, many difficulties are encountered. For this purpose, Ostwald[2] and Hess[3] have devised glass instruments based upon the use of capillary tubes. There is little doubt but that Poiseuille's[4] method of measurement of the rate of flow through a tube, although perfect and very accurate in the case of pure liquids and crystalloid solutions, may yield doubtful results when applied to colloidal solutions. Moreover, fine capillaries are very difficult to clean, and it is probable that the readings are not altogether independent of the pressure applied to the liquid. For these reasons, and because reliable measurements of viscosity of serum were sought, a new instrument was designed to overcome these deficiencies, and to give a high degree of accuracy with a quantity of liquid smaller than 1 cc.

The principle chosen was that of the coaxial cylinders. This principle has been used and studied since 1847 by de Saint-Venant,[5]

[1] Bingham, E. C., Fluidity and plasticity, New York, 1922. The term "viscometer" has been adopted, as proposed by Bingham, as this seems more logical and simple.

[2] Ostwald, W., Manuel pratique de mesures physico-chimiques, Paris, 1904, 291.

[3] Hess, W. R., *Arch. ges. Physiol.*, 1920, clxxx, 61.

[4] Poiseuille, J. L. M., *Compt. rend. Acad.*, 1842, xv, 1167.

[5] de Saint-Venant, unpublished memoirs, February 15, 1847; *Compt. rend. Acad.*, 1872, lxxiv.

Boussinesq,[6] Stokes,[7] and Margules,[8] but mainly by Couette,[9] who made a very careful investigation of the theoretical and practical side of the problem.

The main advantage of the principle of the coaxial cylinders is the direct proportionality between the torque, due to the friction exerted by the fluid on the inside cylinder (if the outside cylinder is rotating), and the rate of rotation. In other words, the readings are proportional to the rate; hence great facility for changing the sensitivity and increasing the range is obtained. This has been well established by Couette, who has shown that up to a certain maximum the ratio $\frac{P}{N}\left(\frac{\text{torque}}{\text{No. of revolutions}}\right)$ is constant. In his instrument, the radii of the cylinders of which were respectively 14.63 cm. and 14.39 cm., the maximum was 55 R. P. M., which corresponds to a linear speed of 4,840 cm. per minute. When this limit is attained, the proportionality no longer exists, owing to turbulent régime. For the viscosity of air, he shows that the same ratio (expressed this time as $\frac{T}{N}$, T being the angle of torsion) is constant within 0.01 per cent up to 300 R. P. M. As in our instrument the average number of revolutions (for liquids such as water, alcohol, or sera, *i.e.* between $\eta = 0.003$ and $\eta = 0.0200$) is always less than 16 R. P. M., which means a linear speed of 50 cm. per minute, it is obvious that no error can arise from turbulent régime.

The formula used by Couette, which he obtained by integrating the equations of Navier,[10] is

$$M = \frac{4\pi\Omega R_1^2 R_0^2 h\epsilon}{R_1^2 - R_0^2}$$

Ω = angular speed of external moving cylinder of radius R_1.

M = momentum, with respect to the axis, of the friction exerted by the fluid on a slice of height, h, in the inside cylinder of radius R_2.

---

[6] Boussinesq, Essai sur la théorie des eaux courantes, Paris, 1873, 62.

[7] Stokes, *Tr. Cambridge Phil. Soc.*, 1845, iii, 287.

[8] Margules, *Wien. Ber., 2te Abt.*, 1881, lxxxiii, 588.

[9] Couette, *J. chim. et physique*, series 6, 1890, xxi, 433. *J. physique*, series 2, 1890, ix, 414.

[10] Navier, *Mém. Acad. sc.*, 1823, vi, 389.

ϵ = coefficient of internal friction. (The generally accepted symbol is now η.) This formula expresses, in the case of his instrument, the value of the coefficient of viscosity, ϵ or η.

The two principal objections to this method are, first, the sliding of the liquid on the surface of the cylinders, and, second, the errors arising from the eccentricity of the two cylinders. We shall review them briefly.

Sliding.—This question has been discussed ever since the problem was stated, and the defenders of the opinion that the liquid slides have been unable so far to base their statements upon experimental evidence. The very careful experiments of Couette led him to conclude that there was no sliding at all, whatever the walls of the cylinders were made of. This is also the opinion of Bingham,[1] who quotes many others. The experiments of Brodmann,[11] who holds the opposite opinion, do not seem very convincing, since the variation of temperature, during one experiment, reached 0.4°C., and his apparatus was not as perfect theoretically as that of Couette.

Eccentricity.—Couette has made a careful study of the part played by the eccentricity and has obtained the formula

$$M = \frac{2\pi\epsilon\Omega R_0^2 R_1}{a}\int_{z_0}^{z_0+h}\frac{dz}{\sqrt{1-m^2}}$$

$a = R_1 - R_0$, mean thickness of liquid layer.
m = relative eccentricity ($o > m > 1$).

This leads to the following conclusions, the first of which is of capital importance.

(a) The eccentricity does not affect the proportionality of the momentum, M, to the rate of rotation and to the coefficient of viscosity.

(b) The momentum, M, is minimum when $m = o$; that is, when the cylinders are coaxial.

(c) If the eccentricity is neglected, a very small relative error by excess is introduced, provided the centering has been realized approxi-

[11] Brodmann, C., *Ann. Physik. u. Chem.*, 1892, xlv, 159.

mately. If the axes are parallel and the eccentricity = m, the relative error in the evaluation of e will be

$$\frac{m^2}{2}$$

m being smaller than 1, the error is small. Moreover, it does not affect the relative value of measurements made in series, for, as stated under a the readings will be comparable.

Description of the Instrument.

The pictures show at a glance the general disposition of the first model (Figs. 1 and 2). The rotating cup ordinarily used has an inside diameter of 10 mm. and the diameter of the plunger is 8 mm. 0.8 to 1 cc. of liquid covers the plunger entirely when immersed in the cup (Fig. 3). The phosphor-bronze suspension is a standard Leeds and Northrup galvanometer strip 0.002 inches in diameter, approximately 144 mm. long. In our first model, the plunger was removed by unscrewing it from the piece supporting the mirror. This will be improved upon, and the plunger will simply be hooked to its support. Behind the mirror is a light aluminum wire, carrying an aluminum foil vane, which acts as a damping device by moving in a semicircular bath filled with paraffin oil. This device is sufficient to insure a steady spot. By replacing the scale by a recording photographic camera, as used with the Einthoven galvanometer (cardiograph), a record in function of time or temperature may be obtained.

This first model was built with a fixed oil bath inside of which is the revolving table; the suspended system is mobile and can be raised by means of rack and pinion, in order to permit the cleaning, filling, and changing of the cup. Another improved model will be built, on the contrary, with a fixed suspended system, the rotating cup and its bath being so disposed as to slide out of the way. This will prevent the jarring of the suspension and insure a more stable zero position.

The bath is filled with oil. A resistance is immersed in it so as to raise its temperature by means of an outside rheostat when a constant increase of temperature is required. In order to keep the oil bath at a constant temperature different from that of the room, it was built

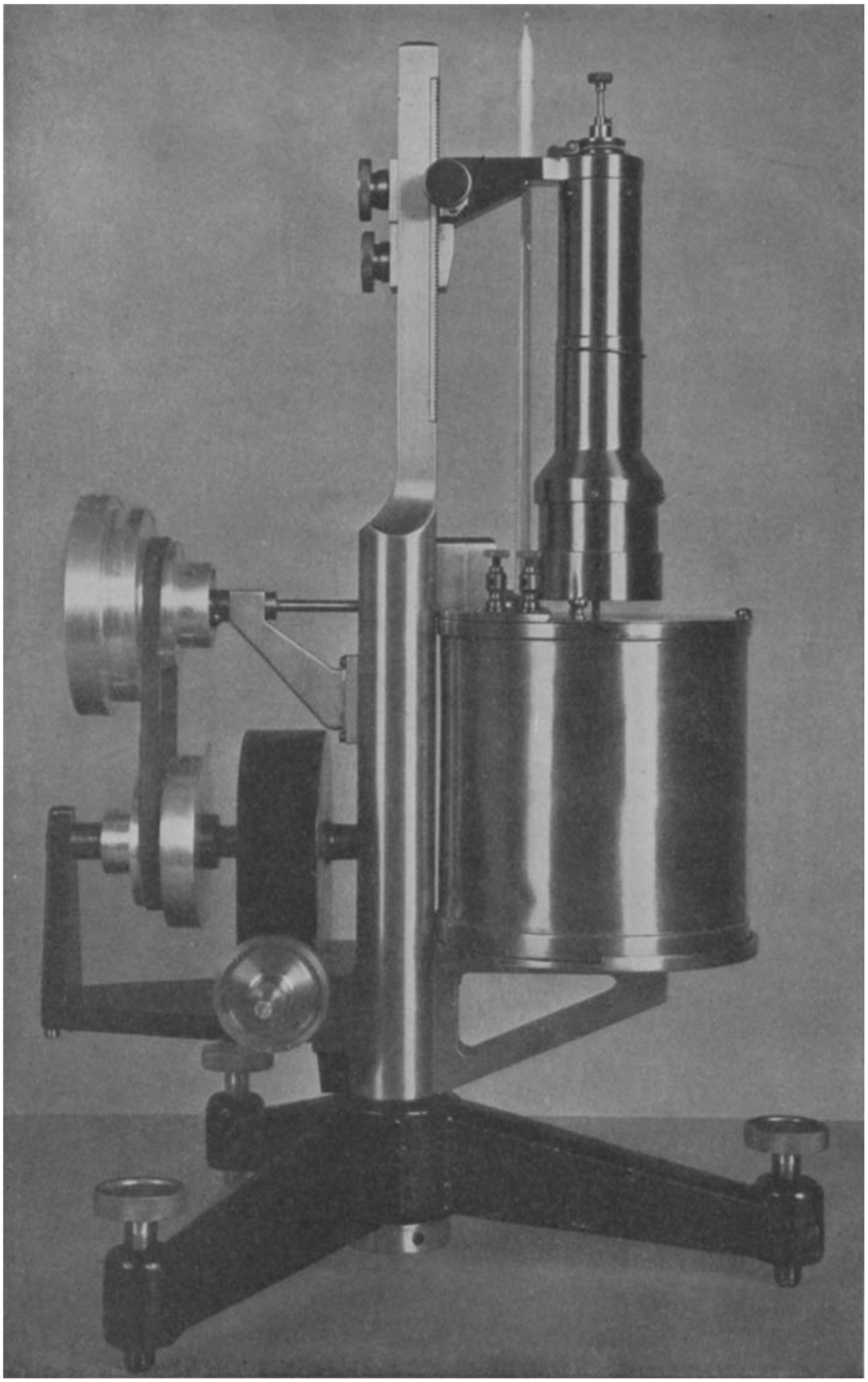

FIG. 1. View of the viscometer, mounted.

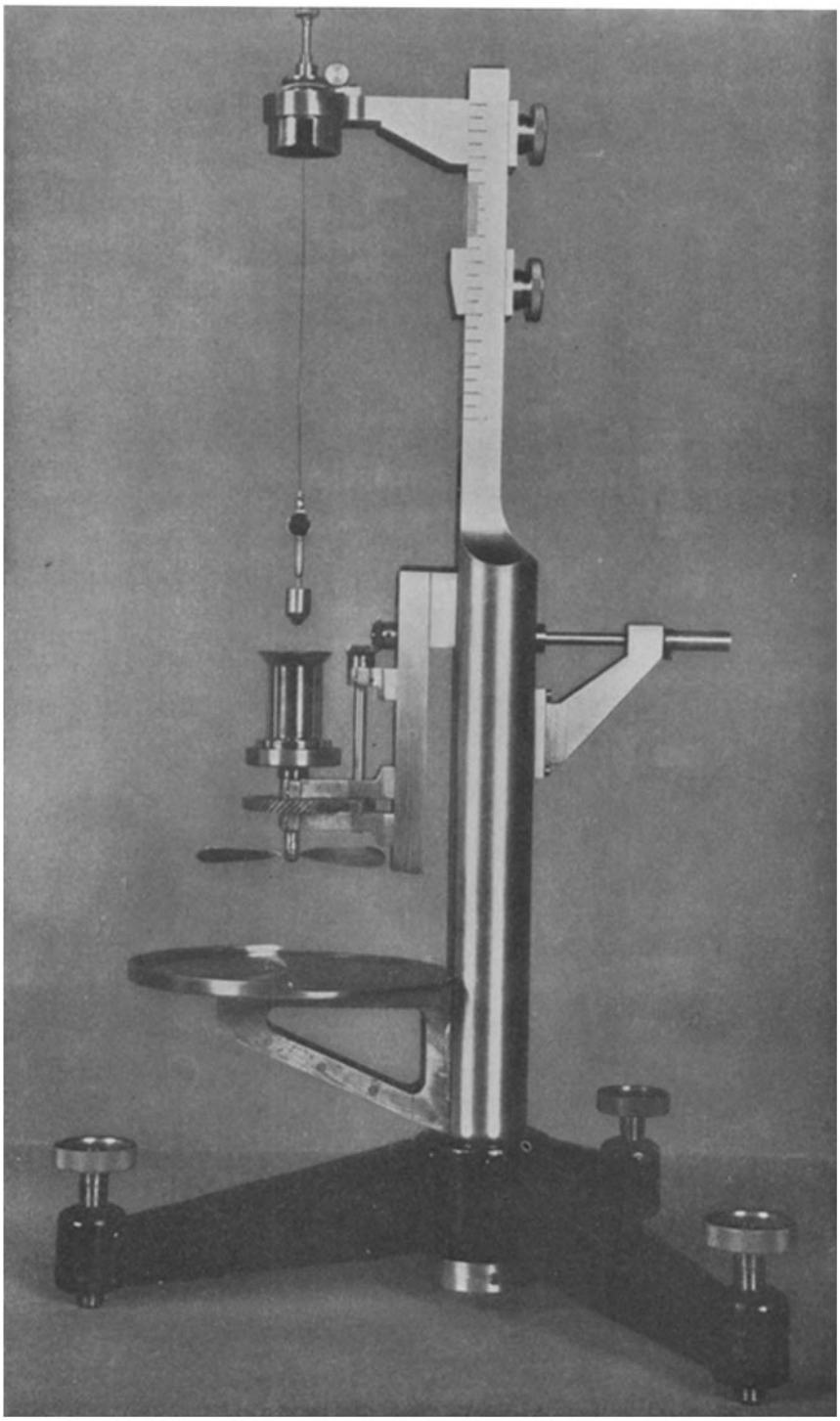

FIG. 2. Details of the suspension and rotating cup.

with double walls and a flow of water may be made to circulate between them. The temperature of the running water is regulated outside by means of any good temperature regulator, within 0.1 of a degree C. Thus, the oil bath acts as a thermic fly-wheel, and the changes of temperature in it do not amount to more than 0.01 of a degree, because the thermic inertia of the mass of oil tends to flatten the oscillations of temperature of the water, as there is no synchronism between the two. It was found convenient to heat the inflowing water by means

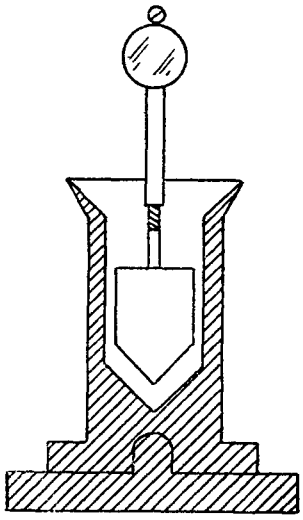

FIG. 3.

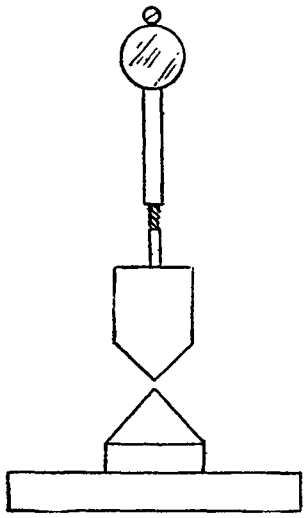

FIG. 4.

FIG. 3. Relative position of plunger and cup.

FIG. 4. In order to secure the centering of the plunger, a cone may replace the cups so that the two summits are brought into coincidence by means of the leveling screws.

of an ordinary immersion heater, and to keep the same water running through the heater, the thermoregulator, and the viscometer by means of a gear pump. The motor which keeps the cup in motion must be a constant speed motor. A constancy of 0.25 per cent is sufficient. Such motors, with speed regulators, are on the market.

In order to center the plunger and rèduce the eccentricity to a minimum, a cone is placed on the revolving table instead of the cup

(Fig. 4). Then the plunger is lowered until the summits of the two cones come almost in contact. It is then very easy, by means of the leveling screws, to obtain a satisfactory coincidence of the axes. As both cones are nickel or gold plated, and highly polished, the light is reflected by them in the shape of very sharp white triangles which, when the coincidence is good, seem to be produced by two cross lines. The diameter of the threaded "tail" from which hangs the plunger is only 1 mm., which is very small with respect to the plunger, so that a slight difference in the level of the liquid in the cup, provided the top of the plunger is covered, does not affect the readings. It is preferable to use the same amount always, but it is sufficient to measure it within 0.05 cc.; the error arising from the increased surface of friction, being of the order of magnitude of the 9th decimal, cannot be detected.

Standardization and Use.

Thus equipped, the instrument may be standardized from the reading obtained with air. The viscosity of air is very well known. The formula given by Sutherland[12] in 1893 expresses accurately the variations in function of temperature of the viscosity of gases.

$$\eta = \eta_o \sqrt{\frac{\Theta}{273}} \frac{1 + \frac{C}{273}}{1 + \frac{C}{\Theta}}$$

where

Θ = absolute temperature (θ + 273).
η_o = coefficient of viscosity at 0°C.
η = coefficient of viscosity at θ°C.
C = Sutherland's constant (equal to 119.4 for air).

The viscosity of air at 0°C. is 0.0001733; at 24°C., it is 0.0001852; at 25°C., 0.0001858; at 26°C., 0.0001865. The temperature can be measured very accurately so that no possible error may arise from this fact. With our instruments we obtained, for a given place of the plunger in the cylinder, a deviation of 5.5 mm. on the scale at 1 m., the cup rotating at 16 R. P. M., and 109.5 mm. when the cup rotated at 320 R. P. M.; *viz.*, 20 times faster. In order to obtain closer accuracy,

[12] Sutherland, quoted by Brillouin, M., Recueil de constantes physiques, Paris, 1913, 105.

109.5 is divided by 20, thus yielding 5.48 instead of 5.5. On the scale it is impossible to make the difference between 5.5 and 5.48. Under the same conditions, water (at 16 R. P. M.) gave 264 mm. A simple ratio yields the value of η for water at the given temperature.

$$\frac{264 \text{ (reading for water at 25°C.)}}{5.48 \text{ (reading for air at 25°C.)}} = \frac{x \text{ (viscosity of water)}}{\eta_a \text{ (viscosity of air)}} = \frac{0.008912}{0.000185}$$

0.00891 is the value obtained by the capillary tube (Poiseuille's) method as given, among others, by Thorpe and Rodger[13] in 1894. We find 0.008912. The 6th decimal may be neglected. In ordinary practise, the proportion given above may be solved readily on the slide rule and with sufficient accuracy, since 0.01 of a degree C., between 20 and 30°C., brings forth a change of 0.00002 in the value of the coefficient of viscosity (which corresponds to a relative error of 0.2 per cent); hence, the 5th decimal may only be relied upon in case the temperature can be controlled within 0.01 of a degree, and this, of course, can only be realized under special conditions. The instrument was taken apart, and when it was again ready for use, an experiment was made under very different conditions. The plunger was lowered in order to reduce the space between the bottom of the cup and the plunger, thus increasing the sensitivity. The following figures were obtained.

Temperature, 24.95°C. (oil bath).
No. of revolutions of the cup, 5 per minute.
Reading for air (on the scale), 3.9 approximately (first reading).
No. of revolutions of the cup, 30 per minute (six times faster).
Reading for air, 23 divisions $\div$ 6 = 3.833 (second reading)
Coefficient of viscosity of air at 24.9 = η_a = 0.0001857.
Reading for water (0.9 cc.), (cup rotating at 5 per minute), 182 divisions.
Hence, $\frac{182}{3.833} = \frac{\eta_w}{0.0001857}$, $\eta_w = 0.008820$.

The previous reading for water was $\eta_w = 0.008912$ at 25°C. Thus, the difference is 0.000092. The difference in temperature was 0.05°C., which should bring forth a difference of 0.0001 in the coefficient. Hence, there is a discrepancy of 0.000008 units C. G. S., which means

[13] Thorpe and Rodger, quoted by Gouré de Villemontée, G., Recueil de constantes physiques, Paris, 1913, 109.

a relative error of 0.1 per cent. As an error in the estimation of the temperature of 0.01 of a degree introduces an error of 0.2 per cent, it is obvious that this limit is an optimum which cannot be improved upon, as long as the temperature is only known within 0.01°C.

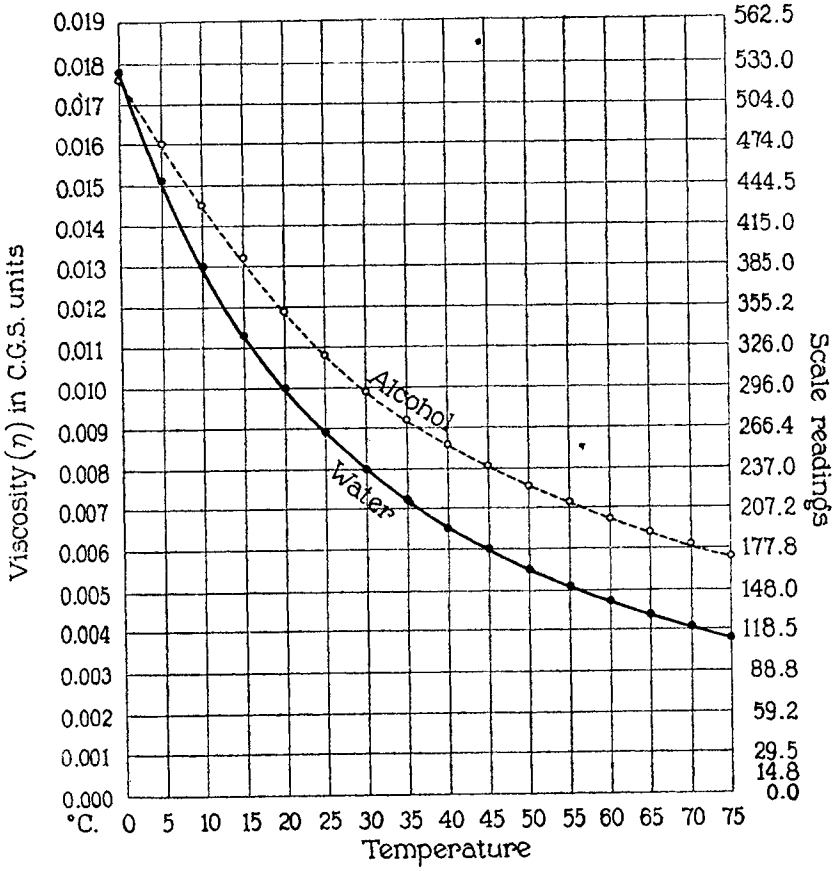

FIG. 5. Viscosity of water and alcohol in function of temperature. The values obtained with the viscometer are represented by the curve. The standard values published by Thorpe and Rodger (Poiseuille's method) are expressed by the solid black circles.

By increasing the speed of rotation and by using a larger plunger, even with the same suspension wire, a deviation of 500 divisions (full scale) may easily be obtained with air. Thus the viscosity of gases can be measured to the 7th decimal figure.

The instrument is connected with the motor by means of a set of inverted pulleys of accurately known ratio. Unless the viscosity of very viscous substances is sought, the ratios 1, 2, 6, and 12 were found to give a large range, sufficient for the average laboratory (roughly from 0.0001 to 0.1, or from 0.01 up to 10 times the viscosity of water).

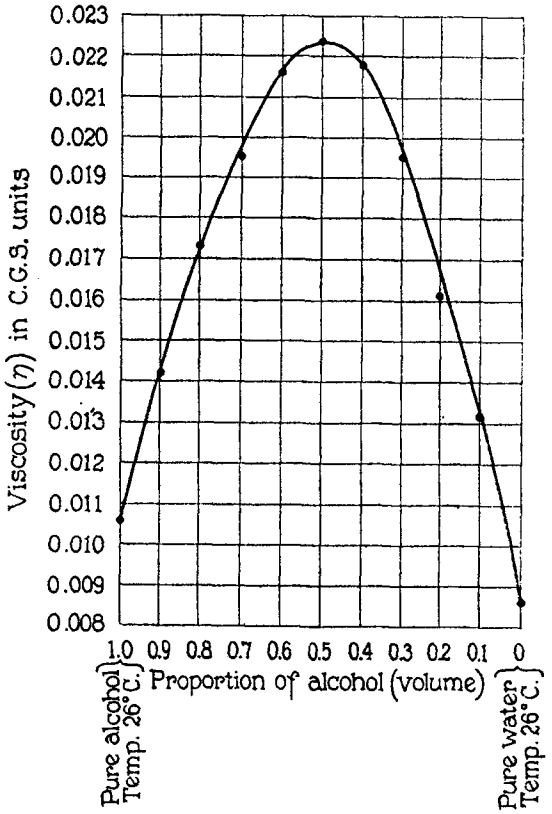

FIG. 6. Viscosity of a mixture of water and alcohol at 26°C.

To extend the range, it is only necessary to change the ratio by choosing another set of pulleys. In order to control the ratio exactly, flat pulleys are used instead of grooved ones. Gears were avoided as they are much more complicated when it comes to changing the ratio, more noisy, and do not always run as smoothly. A flat rubber belt was found to be quite satisfactory.

The coefficient of viscosity in units C. G. S. is not as frequently used in physiological laboratories as the ratio $\frac{\eta}{\eta_w}$ which may be termed specific viscosity, η being the coefficient of viscosity of the liquid (proportional to the reading on the scale) and η_w the coefficient of viscosity of water at the same temperature. This last figure may be measured with the instrument or it may be taken from the curve (Fig. 1).

Before using the viscometer for other liquids, it is advisable to measure the viscosity of water or absolute alcohol. It will be found that, on account of the smallness of the cup and the high value of the surface tension of pure water, its viscosity is more difficult to measure than that of alcohol. In order to make it very easy, it is only necessary to touch the surface of the water with a piece of wire previously dipped in oil (paraffin oil, for instance). A very thin layer of oil spreads over the water, decreasing the surface tension without changing the viscosity in the least, and the measurements are just as accurate as when alcohol is used. Fig. 5 gives the curve of the variations of the viscosity of water and alcohol in function of temperature. Fig. 6 gives the viscosity of a mixture of water and alcohol at 26°C.

CONCLUSIONS.

A viscometer is described, particularly adapted to the measurement of colloidal solutions and physiological liquids (serum, blood), which, although doing away with the capillary tube (which is difficult to handle and to clean), requires less than 1 cc. of liquid.

The readings are rapid, from 30 seconds to 1 minute. The temperature is easily controlled and varied. The range is considerable and may be changed easily. The sensitivity is such that the viscosity of air can be measured to the 7th decimal, without any difficulty.

Its main advantage is that it makes possible the observation of the changes occurring in the viscosity of a liquid in function of time and temperature. It may be used as a recording viscometer by simply placing a recording photographic box before the mirror.