THE COLOR VISION OF DICHROMATS

II. SATURATION AS THE BASIS FOR WAVELENGTH DISCRIMINATION AND COLOR MIXTURE

BY SELIG HECHT AND SIMON SHLAER

(From the Laboratory of Biophysics, Columbia University, New York)

(Accepted for publication, November 27, 1935)

I

Wavelength Discrimination as Saturation

The three variables of color vision are hue, saturation, and brightness. The characteristic color differences along the spectrum which the color-normal sees are essentially concerned with hue. In measurements of wavelength discrimination the precise value of the just perceptible difference in wavelength at constant brightness is no doubt influenced by saturation differences, but the major factor is hue. What determines wavelength discrimination in the colorblind? Since the best λ discrimination of protanopes and deuteranopes occurs near 500 mμ, which is a region matched with white, saturation may be a more important factor for them than for the color-normal.

A simple but striking experiment demonstrated to us that saturation is indeed the determining factor.[1] One side of the field in the Helm-

---

[1] Actually we arrived at this notion entirely from theoretical considerations. Adopting the quantitatively formulated trireceptor idea of Young (1807) as a basis for color vision (Hecht, 1930; 1934) and supposing that in colorblindness one of the cone primaries, say the red, is transformed into one of the other two, say the green, then three things follow. First, near 500 mμ all three curves will intersect in one point; the resulting sensation will be white and will correspond to the neutral point. Second, to the left of this neutral point, where the blue primary is higher than the other two, the sensation will be (*a*) white by virtue of those effects where the blue, green, and red primaries have the same height, and (*b*) blue by virtue of the excess effect of the blue primary. Different portions of the spectrum between the neutral point and the short-wave end will then differ only in the relative amounts of blue and white, that is, in saturation. Third, on the right side of the neutral point, the now similar red and green primaries are higher than the

holtz Color Mixer is set for 520 mμ (green to us) while the other side is set for 650 mμ (red). The colorblind cannot match these two fields merely by adjusting their brightness differences. However, when a little white light is added to 650 mμ, the colorblind at once reports the two fields to be very nearly matched; and further slight additions of white remove all differences between the two fields to the colorblind, though to us they are almost as widely different as before. The same experiment may be made with 480 (blue-green) and 420 mμ (violet), but it is not so striking because these two wavelengths are not so sharply different to us as red and green.

Though we made this discovery quite independently on the basis of theoretical argument, examination of the literature showed that the phenomenon had already been found by von Kries and Küster (1879). However, no quantitative investigation has ever been made of it. We therefore measured the situation throughout the spectrum with our protanope H. J. in the hope of supplying a new type of data for the color vision of the colorblind.

II

Procedure

For matching the short-wave side of the spectrum, we used mixtures of 440 mμ with white of 5000°K. For the long-wave side, we used 650 mμ with the same white, for which on some occasions we substituted the white of the neutral point at 491.6 mμ.

The procedure was essentially the same as with measurements of λ discrimination described in the preceding paper. A mixture of white and 440 mμ or of white and 650 mμ was placed in half the field of the Helmholtz Color Mixer. In the other half, light of a given wavelength was placed, and the subject reported whether he could match the two fields merely by varying the brightness of this light alone. Successive wavelengths were tried until the values of λ_1 and λ_2 were located between which the subject could match the mixture of 440 mμ (or of 650 mμ) and white in the other half of the field. We found it convenient to locate the two edges of this matching band by working from non-matching regions on each side to the matching region between them. The readings were clearer than by

---

blue. Everything below the blue curve represents white; everything between the blue curve and the identical red and green curves is yellow. The different wavelengths then produce merely different relative effects of yellow and white; again this means differences in saturation only.

working steadily across the matching region until reaching the non-matching portion; this has already been referred to in the preceding paper.

A series of known mixtures were measured in this way, covering the spectrum. We always set the mixture and the wavelength of the spectral light, while the subject controlled the brightness of the spectral light and made the judgment of match or no match.

In order to record the data quantitatively it was necessary to determine the relative brightnesses of the 5000°K white, of the neutral point white, of 440 mμ, and of 650 mμ. These were measured by H. J. himself. The 5000°K white light was put into the left half of the field, and homogeneous light of the neutral point (491.6 mμ) into the other, and their relative brightness measured. The whole white light was then removed, and 491.6 mμ from the right collimator placed in its stead and its brightness measured in terms of the 491.6 in the other field. From this we found the relative brightness of the white and the 491.6 on the same side of the field. The relative brightnesses of 491.6, 440, and 650 mμ as they occur in the Color Mixer were taken from H. J.'s measurements of brightness distribution in the spectrum already recorded in the preceding paper. Knowing these values, we were able to make mixtures corresponding to any desired brightness ratio.

III

Saturation Distribution

The data are in Table I, and record the two positions in the spectrum, λ_1 and λ_2, between which H. J. was able to match the specific mixtures of white and 440 mμ or of white and 650 mμ given in terms of the brightness ratio of color to white. Those mixtures which were made with 650 mμ plus the neutral point (491.6 mμ) instead of whole white light, are printed in italics. The four measurements in parentheses were made by the method of working through the matching band rather than from both sides of the band.

The best way to grasp the meaning of the data is to examine Fig. 1 in which λ_1 and λ_2 are plotted against the logarithm of the brightness ratio of the mixtures, and to compare this figure with Figs. 2 and 6 of the preceding paper. There is at once evident a close correlation between these data and those of λ discrimination and of color mixture. The horizontal distance between the pairs of lines to the left and right of the neutral point is really a measure of $\Delta\lambda$ (including two discrimination steps), because a mixture which matches a given λ will naturally also match those values of λ which fall within the band $\Delta\lambda$ previously found. In Fig. 1 the $\Delta\lambda$ band is narrow near the neutral point; to the

TABLE I

Limits (λ_1 and λ_2) of Spectral Bands Matched by Protanope H. J. with Mixtures of White Light Plus Either 440 mμ or 650 mμ

440 mμ and white			650 mμ and white		
Brightness ratio 440/white	λ_1	λ_2	Brightness ratio 650/white	λ_1	λ_2
70.8		473.3	166.0	572.9	
36.3		472.5	129.0	573.0	637.7
17.8		475.2	110.0	569.0	
9.12		473.2	79.4	560.1	
5.50		474.0	55.0	545.2	
3.16		473.6	43.7	563.2	629.0
1.59		474.0	38.9	538.6	
1.59		474.0	*38.0*	*525.3*	
0.759		475.7	*27.5*	*521.0*	
0.741	424.6	(467.4)	26.9	524.4	
0.380		476.1	26.9	526.0	
0.372	443.3	(466.4)	*20.9*	*523.1*	
0.324		478.0	20.4	517.3	554.0
0.246	459.0	(470.7)	15.5	515.0	
0.246		478.8	*13.5*	*515.4*	
0.246		480.4	10.7	512.0	524.9
0.135	464.6	(474.7)	*9.33*	*511.2*	
0.135		478.0	8.91	510.7	513.8
0.0589	473.1	483.0	*5.50*	*507.3*	
0.0589		480.4	4.37	506.0	509.2
0.0589		483.3	4.37	504.2	506.3
0.0219	482.8	486.0	*3.16*	*502.7*	*503.6*
0.0219		486.0	2.95	501.3	503.3
0.00977	486.0	488.8	2.19	499.8	500.8
0.00407	488.8	489.7	*1.91*	*500.8*	*501.0*
0.00148	489.6	491.3	*1.26*	*496.5*	*497.4*
			0.977	495.9	497.8
			0.724	495.6	497.0
			0.589	*493.5*	*494.3*
			0.316	491.3	493.3
			0.295	*492.9*	*493.5*
			0.295	492.2	493.3
			0.138	*491.6*	*492.6*
			0.107	488.8	490.7
			0.0851	491.2	493.0
			0.0550	*491.2*	*493.5*
			0.0302	*490.7*	*491.6*

left it widens rapidly, while to the right it spreads only slowly up to about 570 mμ, beyond which it also widens rapidly. This is precisely what happens to Δλ for H. J. in the wavelength discrimination measurements shown in Fig. 2 of the preceding paper. Notice that the region between 470 and 570 mμ is one of sharply changing saturation for the colorblind, and that this same region is one of high wavelength discrimination. Since at the neutral point the spectrum appears white and therefore completely unsaturated, the change of the spectrum to either side of the neutral point must be in the direction of increasing saturation, the maximum saturation being reached at the two ends.

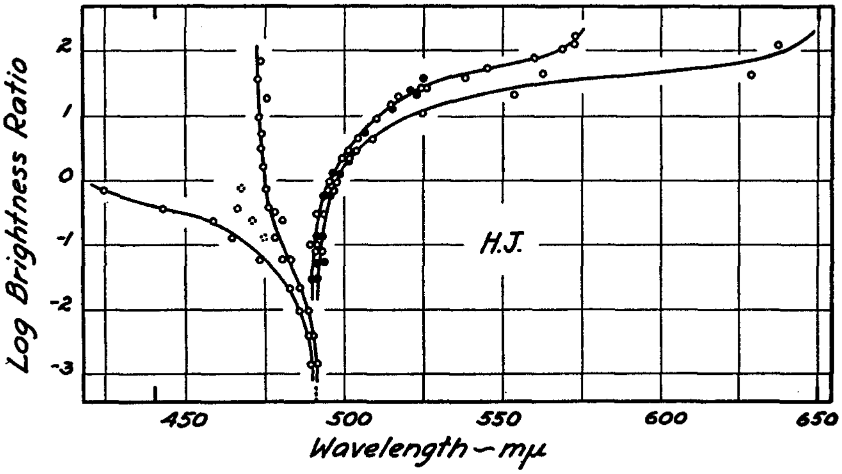

FIG. 1. Relative saturation in the spectrum for the protanope H. J. Mixtures of 440 mμ and white are to the left of the neutral point; mixtures of 650 mμ and white are to the right. The points give the limits of the band Δλ which H. J. matches with each particular mixture. To the right of the neutral point, the mixtures using white of color temperature 5000°K are open circles, while those using the white of the neutral point (491.6 mμ) are solid circles. No difference is apparent in the results. The dotted circles are the ones in parentheses in Table I and were secured by working continuously across the matching band from left to right instead of from each side.

Some estimate of the degree of saturation at the two ends of the spectrum may be made in terms of the data. Fig. 1 shows that to match a just perceptible step from the white of the neutral point, it is necessary to have a mixture of one part of 440 mμ to about 100 parts of white. This means that H. J. can tell the difference between white, and white containing about 1 per cent of 440 mμ. The least perceptible colorimetric purity of 440 mμ is therefore 0.01 for the colorblind. Priest and Brickwedde (1926; see their data in Hecht, 1932) give 0.0022 as the average least perceptible colorimetric purity of 440 mμ for their normal eyes. Assuming the saturation of a spectral color to be inversely proportional to its least perceptible colorimetric purity, then 440 mμ appears about five times as saturated to the normal eye as it does to the colorblind eye.

A similar computation may be made for 650 mμ. H. J. can just differentiate the neutral point from a mixture of white containing about 10 per cent of 650 mμ. This gives 650 mμ a least perceptible colorimetric purity of about 0.1. Priest and Brickwedde's average for this wavelength is 0.0059 which makes 650 mμ about twenty times as saturated for the normal as for the colorblind.

Evidently, the spectrum for the colorblind is reduced in saturation to different degrees in its different parts; so much so that at the neutral point it is completely unsaturated.

It is worth pointing out a curious paradox in this connection. From Fig. 1 it can be seen that the addition of 10 per cent white to 650 mμ enables H. J. to match it with about 520 mμ, a really large shift in λ. On the other hand, the addition of 10 per cent white to 440 mμ produces hardly any change in the λ which it will match; to match 440 mμ with 470 mμ—a comparatively small shift—requires the addition of 10 parts of white to 1 part of 440 mμ. Thus the protanope H. J. is much more sensitive to the addition of white to 650 mμ than to 440 mμ. If the addition of white to a saturated color is more easily perceptible than the same addition to an unsaturated one, this means that 650 appears more saturated to H. J. than does 440 mμ. However, judging by the least perceptible colorimetric purity computation, 440 is about ten times as saturated as 650 mμ.

This apparent contradiction is one of interpretation only. It may mean that the colorblind is more sensitive to changes in an unsaturated color than in a saturated one, or that least perceptible colorimetric purity in the colorblind is no measure of saturation, or something different from either. It leaves untouched the main point to be made from the present measurements, which is that saturation is the basis on which the colorblind discriminates wavelength on either side of the neutral point.[2]

IV

Color Mixture as Saturation

We have referred to the gauging of the spectrum with two primaries by the dichromat as color mixture. It is important to point out that this is strictly incorrect; that, just as the basis of wavelength discrimination is not hue (that is, color) but saturation, so the basis of spectrum gauging is also not hue but saturation.

The dichromat cannot distinguish his neutral point from white. Moreover, a specific mixture of two primaries also matches the neutral point. Therefore all other mixtures of the two primaries consist of two parts: one part made up of the whole of the first primary plus the necessary fraction of the second to make white, and the other part made up of the excess of the second primary. To the left of the neutral point, the short-wave primary dominates and the variation along the spectrum consists merely in the ratio of its luminosity to that of the associated white made up of the neutral point mixture, while to the right of the neutral point the long-wave primary similarly dominates and the spectrum there is matched by the variation in relative amount of primary and white. The situation is precisely the same as that just presented in which the spectrum on either side of the neutral point was matched with mixtures of white and either 440 mμ or 650 mμ. In the case of the color mixture data the two primaries happen to be 458.7 and 570.0 mμ, and the white is made of a mixture of the two equivalent to the neutral point.

If this reasoning is correct, it should be possible to derive from the spectrum gauging (color mixture) data of Table III and Fig. 6 of the preceding paper, the amount of white and of excess primary for each mixture data, and a plot of the ratio of primary luminosity to white luminosity for the spectrum should yield a set of curves very similar to Fig. 1 in which the relative saturation was measured directly.

---

[2] Because saturation is so essential a factor in determining λ discrimination for the colorblind, it is important to investigate its contribution to similar measurements for the color-normal.

The computations involve several steps, and may be best explained by an example. Take the primary brightness ratio 22.5/1 whose matching limits for H. J. are 446.6 and 469.1 mμ. When the brightness of λ570.0 is 1, the total brightness of this mixture is 23.5. The ratio of the same primaries to match the neutral point at 491.6 mμ is 0.063/1, so that when the brightness of λ570.0 is 1 the total bright-

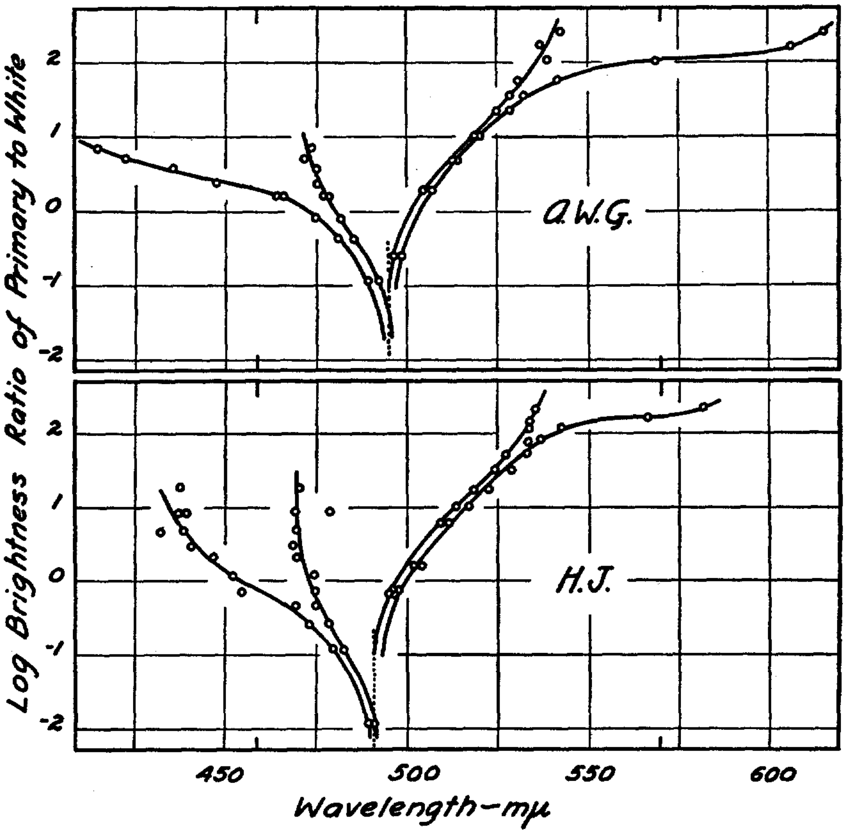

FIG. 2. Relative saturation in the spectrum computed from the color mixture data in Table III and Fig. 6 of the preceding paper. A. W. G. is a deuteranope; H. J. a protanope. Compare this figure with Fig. 1. Even though the primaries (shown as short vertical lines) in the two figures are different, the computed saturation distribution resembles the measured saturation so strikingly as to leave little doubt of their essential identity.

ness is 1.063 and is white. Since the 22.5/1 mixture and the neutral point mixture of 0.063/1 both contain 1 part of 570.0 mμ, the 22.5/1 mixture therefore contains 1.063 parts of white, while the rest, 22.4 parts, is 570.0 mμ alone. The fraction 22.4/1.063 therefore gives the ratio of primary to white, and is the information required.

We have made these computations for the mixture data of H. J. in Table III of the preceding paper, and the results are incorporated in Fig. 2. The results for A. W. G. are also shown in Fig. 2. If Fig. 2 is compared with Fig. 1 showing the saturation distribution as actually measured, it is obvious that the two are of the same form and appearance and yield the same information. The two are not identical because the primaries used are not the same; in Fig. 1 they are 440 and 650 mμ, while in Fig. 2 they are 458.7 and 570.0 mμ. But in spite of this the quantitative similarities are so apparent as to leave scant doubt that on each side of the neutral point the property which determines color mixture for the colorblind is the same which determines wavelength discrimination, and is saturation.

It is worth emphasizing this point because it illustrates the fact that properties of color vision which for the color-normal are independent, become non-independent for the colorblind. In the present case, mixture data and saturation distribution data were secured separately and by an independent technic. Nevertheless, they turn out to be related so that one may be derived from the other. Very likely least perceptible colorimetric purity is a property which is also derivable from the same data.

We began our work with the recognition that the quantitative determination of only four independent conditions is necessary in order to describe the color vision of colorblinds and to derive the spectral distributions of the basic cone primaries. Considering that five or six independent conditions were known for color-normals, it seemed a simple matter to find four for colorblinds. However, the four are not so easily forthcoming for the reason that several conditions which are independent for the color-normal become dependent and identical for the colorblind.

What has come out of the work so far is the recognition of saturation as the factor which determines the quantitative properties of the colorblind in the spectrum on either side of the neutral point. In terms of this, protanopes and deuteranopes see the spectrum as white at the neutral point, shading off on the short-wave side to a color (possibly blue) with decreasing amounts of white in it; and on the long-wave side to a color (possibly yellow) also with decreasing amounts of white in it. The spectrum to the dichromat thus appears made up of only two hues, one at each end; these hues gradually become less saturated in the middle portion of the spectrum, reaching complete unsaturation at the neutral point.

SUMMARY

1. Wavelength discrimination for the colorblind is entirely determined by saturation differences in the spectrum. From the neutral point to the short-wave end, his spectrum may be completely matched by 440 mμ plus white; to the long-wave end by 650 plus white. The proportion of color to white, hence the relative saturation, changes rapidly in the region of small $\Delta\lambda$ at the center, and slowly in regions of large $\Delta\lambda$ at the ends.
2. The data of spectrum gauging with two primaries (color mixture) by the dichromat are shown to contain the saturation distribution in the spectrum for the dichromat. This is because each mixture of primaries may be considered as composed of a mixture which matches white and of an excess of one primary. The data when so computed yield saturation distributions almost identical with those found by direct measurement, and show that on each side of the neutral point the basis of color mixture for the colorblind lies in saturation and not in hue differences.
3. To judge by these measurements, the spectrum for the protanope and deuteranope is composed of only two hues, themselves probably of low saturation, situated one at each end. Toward the center these hues decrease still more in saturation until they completely disappear in the white of the neutral point.

BIBLIOGRAPHY

Hecht, S., The development of Thomas Young's theory of color vision, *J. Opt. Soc. America*, 1930, **20**, 231.

Hecht, S., A quantitative formulation of colour-vision, Report of a Joint Discussion on Vision, Cambridge, University Press, 1932, 126.

Hecht, S., Vision: II. The nature of the photoreceptor process, in Murchison, C., A handbook of general experimental psychology, Worcester, Clark University Press, 1934, 704.

von Kries, J., and Küster, F., Ueber angeborene Farbenblindheit, *Arch. Physiol.*, 1879, 513.

Priest, I. G., and Brickwedde, F. G., The minimum perceptible colorimetric purity as a function of dominant wave length with sunlight as a neutral standard, *J. Opt. Soc. America*, 1926, **13**, 306.

Young, T., On the theory of light and colors, in Young, T., Lectures in natural philosophy, London, Joseph Johnson, 1807, **2**, 613.